P

Dynamics of Uniform Circular Motion

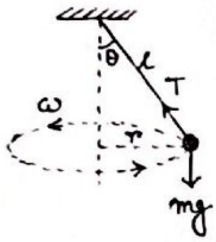
* Centripetal Force: Force necessary to act on a body to rotate it in a circle with a uniform angular velocity.

$$F_{centripetal} = m\frac{v^2}{r} = mv\omega = m\omega^2 r$$

It's not a basic force of nature.
It's a no-work force.

* Centrifugal reaction force: Reaction force due to centripetal force acting away from the centre.

* Conical Pendulum: $T\cos\theta = mg$ | $T\sin\theta = m\frac{v^2}{r}$



i) $\tan\theta = \frac{v^2}{gr}$ ii) $T = m\sqrt{g^2 + \frac{v^4}{r^2}}$

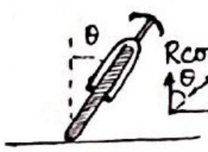
iii) Time Period $= 2\pi\sqrt{\frac{r}{g\tan\theta}} = 2\pi\sqrt{\frac{l\cos\theta}{g}} \approx 2\pi\sqrt{\frac{l}{g}}$ [θ very small]

* Motion of a vehicle on a level circular road:

- Motion of cyclist at a circular turn:

$R\cos\theta = mg$ | $R\sin\theta = m\frac{v^2}{r}$

i) $\tan\theta = \frac{v^2}{gr}$

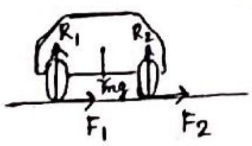


$R\sin\theta$ comes from the static friction of the ground. $f_s = \mu_s R\cos\theta$.

Now, $\mu_s R\cos\theta \geq R\sin\theta \Rightarrow \tan\theta = \frac{v^2}{gr} \leq \mu_s \Rightarrow$ $\boxed{v^2 \leq \mu_s g r}$ – max safe velocity.

- Motion of a car at a circular turn:

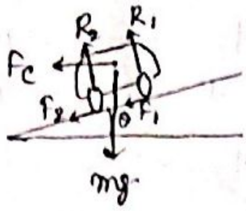
$R_1 + R_2 = mg$ | $F_1 + F_2 = \frac{mv^2}{r}$



$F_1 + F_2 \leq F_s = \mu_s(R_1 + R_2) = \mu_s mg$.

$\frac{mv^2}{r} \leq \mu_s mg \Rightarrow$ $\boxed{v \leq \sqrt{\mu_s g r}}$ max safe velocity.

* Motion of a vehicle on a banked circular road:



$$f_s = \mu_s (R_1 + R_2) = \mu_s R.$$

$$R\cos\theta = mg + f_s \sin\theta.$$

$$R\sin\theta + f_s \cos\theta = \frac{mv^2}{r}$$

$$v_{max} = \sqrt{gr \frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}}$$

* Centrifugal force: Origin of this force is the acceleration of the frame of reference, hence called pseudo force.