Elasticity

* Strain: i) longitudinal, \[ \varepsilon_L = \frac{\Delta L}{L} \]

[\text{N}_0 \text{L}_0 ^{-1} \text{T}^{-2}] ii) Shearing strain, \[ \varepsilon_\theta = \tan \theta = \frac{\Delta \theta}{L} \]

iii) Volume strain, \[ \varepsilon_V = \frac{\Delta V}{V} \]

* Stress: Restoring force/unit area [ML^{-1}T^{-2}]

(a second rank tensor quantity, not a vector).

i) tensile \\ & compressive stress,

ii) shearing stress, iii) volume stress.

* Poisson's Ratio: \( \sigma \), within elastic limit & at constant temperature lateral \\ & longitudinal strain are found to be proportional.

\( 0.25 < \sigma < 0.5 \) \[ \sigma = -\frac{\Delta D/D}{\Delta L/L} \] lateral/longitudinal.

* Hooke's Law: Within elastic limit & at constant temperature, stress \\ & strain. \( \Rightarrow \) stress = \text{Stress} \cdot \text{strain}, \text{Stress} is called modulus of elasticity.

* Moduluses of elasticity:

i) Young's Modulus: \[ Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A \cdot \Delta L} \]

ii) Shear Modulus or Modulus of Rigidity:

\[ G = \frac{F/A}{\theta} = \frac{F}{A \theta} \]

iii) Bulk Modulus: \[ K = -\frac{P}{\Delta V/V} = -V \frac{P}{\Delta V} \]

\( \theta \) compressibility, \( C = \frac{1}{K} = -\frac{1}{V} \frac{\Delta V}{P} \).

* Two kinds (any) of strain always occur together.
Important Relations:

1. \( Y = 3K (1 - 2\sigma) \), \( 2n (1 + \sigma) = Y \)
   (Here, \( \frac{1}{2} > \sigma > -1 \), theoretical).

2. \( \sigma = \frac{3K - 2n}{2n + 6K} \)
   \( \frac{dV}{V} = 2 \frac{dD}{D} + \frac{dL}{L} = (1 - 2\sigma) \frac{dL}{L} \).

3. \( \frac{1}{K} + \frac{3}{n} = \frac{\sigma}{Y} \)

Spring & Hooke's Law: \( F = -kx \)

Equivalent \( K \):

a) Series: \( \frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2} \)

b) Parallel: \( K = K_1 + K_2 \)

c) \( K' = 2K \)

Elastic Strain Energy: \( Q = \frac{YA L^2}{2L} = \frac{YA (AL)^2}{2L} \)

Elastic energy per unit volume, \( q = \frac{Y (AL)^2}{2L^2} \)

\( q = \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times \frac{F}{A} \times \frac{\Delta L}{L} \)

(Valid for all kinds of strain).

Stress - Strain Relation:

A → Proportional limit
L → Elastic limit \( (L \sim \beta) \)
B → Yield point
C → Tensile strength
D → Fracture point.

OB → Elastic deformation  BC → Plastic deformation.

For glass, time to regain its shape after removing stress is quite large. For quartz, phosphor-bronze alloy, silver, gold, this time is too short, hence used for suspension fibres.
Elasticity:

- Elastic hysteresis: (exhibited by vulcanised rubber).
- Non coincidence of forward & backward curves.
- Loss of energy during the whole process as the area of hysteresis loop.
- Materials having large elastic hysteresis are used as vibration absorbers.

- Elastic fatigue: Tiredness after alternating strains, difficult to continue vibrations.

- Perfect rigid - diamond
- Perfect elastic - good quality steel.
- Liquid has no shearing elasticity.

- Elasticity of Gases.
- Isothermal bulk modulus \( (K_T) \), \( K_T = \rho \) - pressure of the gas.
- Adiabatic bulk modulus \( (K_S) \), \( K_S = \rho^\kappa \).

- Change of density when pressure \( \Delta p \) applied:
  \[
  \rho' = \frac{\rho}{1 - \frac{\Delta p}{K}}
  \]