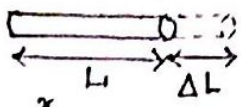
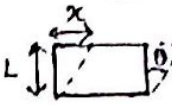
Elasticity

* **Strain** (δ): i) longitudinal, 

$\delta_l = \frac{\Delta L}{L}$

$[M^0L^0T^0]$ ii) Shearing strain, 

$\delta_s = \tan\theta \approx \theta = \frac{x}{L}$.

iii) Volume strain, 

$\delta_v = \frac{\Delta V}{V}$

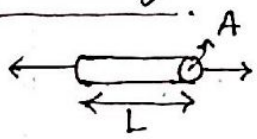
* **Stress** (σ): Restoring force / unit area $[ML^{-1}T^{-2}]$ (a second rank tensor quantity, not a vector).
i) tensile & compressive stress,
ii) shearing stress, iii) volume stress.

* **Poisson's Ratio**: σ, within elastic limit & at constant temperature lateral & longitudinal strain are found to be proportional.

$(0.25 \leq \sigma \leq 0.5)$ $\sigma = -\frac{\Delta D/D}{\Delta L/L}$ | lateral / longitudinal.

* **Hooke's Law**: Within elastic limit & at constant temperature,
stress $\propto$ strain $\Rightarrow$ stress $= M_e \cdot$ strain, M_e is called modulus of elasticity.

* **Moduluses of elasticity**:

i) **Young's Modulus**: 

$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A \cdot \Delta L}$

ii) **Shear Modulus or Modulus of Rigidity**:
$n = \frac{F/A}{\theta} = \frac{F}{A\theta}$. 

iii) **Bulk Modulus**: $K = -\frac{p}{\Delta V/V} = -V\frac{P}{\Delta V}$

⊗ compressibility, $C = \frac{1}{K} = -\frac{1}{V}\frac{\Delta V}{P}$.

* Two kinds (any) of strain always occur together.

* Important Relations:

1. $Y = 3K(1-2\sigma)$, $2n(1+\sigma) = Y$

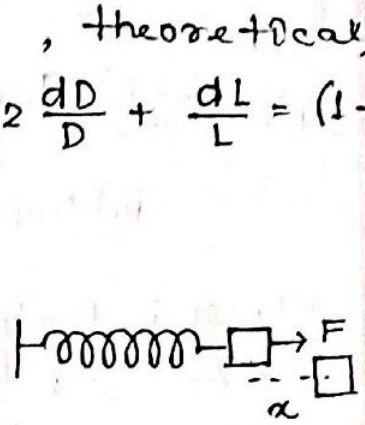
(from here $\frac{1}{2} > \sigma > -1$, theoretical).

2. $\sigma = \dfrac{3K-2n}{2n+6K}$

3. $\dfrac{1}{K} + \dfrac{3}{n} = \dfrac{9}{Y}$.

4. $\dfrac{dV}{V} = 2\dfrac{dD}{D} + \dfrac{dL}{L} = (1-2\sigma)\dfrac{dL}{L}$.

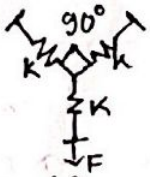
* Spring & Hooke's Law:



$F = -Kx$

equivalent K : a) series - $\dfrac{1}{K} = \dfrac{1}{K_1} + \dfrac{1}{K_2}$

b) parallel - $K = K_1 + K_2$

c) 

$\rightarrow K' = 2K$

* Elastic Strain Energy:

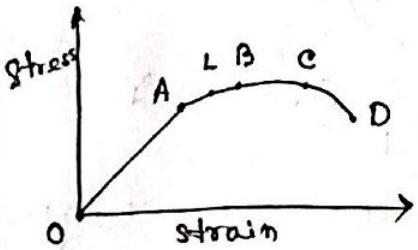
$Q = \dfrac{YAl^2}{2L} = \dfrac{YA(\Delta L)^2}{2L}$.

elastic energy per unit volume, $q = \dfrac{Y(\Delta L)^2}{2L^2}$

$q = \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times \frac{F}{A} \times \frac{\Delta l}{L}$.

(Valid for all kinds of strain).

* Stress - Strain Relation:



A → proportional limit
L → elastic limit (L ~ B)
B → yield point
C → tensile strength
D → fracture point.

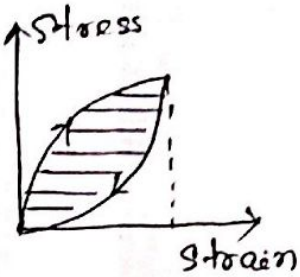
OB → elastic deformation BC → plastic deformation.

BC is lengthier means material is ductile.

* For glass, time to regain its shape after removing stress, is quite large. But, for quartz, phosphorbronze alloy, silver, gold, this time is too short, hence used for suspension fibres.

* **Elastic hysteresis:** (exhibited by vulcanised rubber).

non coinsidence of forward & backward curves.



- loss of energy during the whole process is the area of hysteresis loop.

Materials having large elastic hysteresis are used as vibration absorbers.

* **Elastic fatigue:** Tiredness after alternating strains, difficult to continue vibrations.

* Perfect rigid — diamond
  Perfect elastic — good quality steel.

* Liquid has no shearing elasticity.

* **Elasticity of Gases.**

- Isothermal bulk modulus (K_T), $K_T = p$ = pressure of the gas.
- Adiabatic bulk modulus (K_s), $K_s = \gamma \cdot p$.

* Change of density when pressure (dp) applied:

$$\rho' = \frac{\rho}{1 - \frac{dP}{K}}$$