* Field due to a magnet (Magnet is a dipole):

1. End-on position:
\[ \mathbf{B} = \frac{\mu_0}{4\pi} \frac{2\mathbf{L}}{(r^2 - x^2)^{3/2}} \]

\[ \mathbf{B} = \frac{\mu_0}{4\pi} \frac{2\mathbf{L}}{r^3} \quad [\mathbf{L} = p \times 2L \hat{\mathbf{n}}] \]

2. Broad-on position:
\[ \mathbf{B} = -\frac{\mu_0}{4\pi} \frac{\mathbf{H}}{(r^2 + x^2)^{3/2}} \]

\[ \mathbf{B} = -\frac{\mu_0}{4\pi} \frac{\mathbf{H}}{r^3} \]

3. At a position \((r, \theta)\) in space:
\[ \mathbf{B} = \frac{\mu_0}{4\pi} \frac{\mathbf{H}}{r^3} \sqrt{1 + 3\cos^2 \theta} \]

\[ \tan \phi = \frac{1}{2} \tan \theta \]

* Magnetic Potential Due to Bar Magnet:
\[ \mathbf{V}_p = \frac{\mu_0}{4\pi} \frac{M \cos \theta}{r^2} \]

* \( \mathbf{E} = \mathbf{H} \times \mathbf{B} \), \( \mathbf{U} = -\mathbf{H} \cdot \mathbf{B} \)

* Magnetic moment is more fundamental than magnetic pole strength.

* Magnetic flux:
\[ \Phi_m = \int_S \mathbf{B} \cdot d\mathbf{s} \]

unit: 1 weber = 1 tesla \(\times\) meter\(^2\).

* Gauss's Law in Magnetism:
\[ \oint_S \mathbf{B} \cdot d\mathbf{s} = 0 \]
Earth's Magnetism:

- $N_g, N_m \rightarrow$ Geographic north & Magnetic north
- $S_g, S_m \rightarrow$ Geographic & Magnetic South.
- $E_g, E_m \rightarrow$ Geographic & Magnetic Equator.

Elements of Earth's magnetic field:

1. Declination ($\phi$): Vertical plane passing through $N_g$, $S_g$, & point of observation is geographical meridian. Vertical plane passing through $N_m$, $S_m$, & point of observation is called geomagnetic meridian. Angle between these planes is called declination ($\phi$).

The vertical plane passing through the axis of a freely suspended magnet at its rest position is the magnetic meridian.

Declination at a place is $5^\circ$E means the magnetic & geographical meridian make an angle $5^\circ$E north pole is towards the east.
2. Inclination or dip ($\theta$): The angle the earth's magnetic field vector $\mathbf{F}$ makes with horizontal direction on the magnetic meridian at a place.

Dip at a place is 35°N means if a freely suspended magnetic needle is placed there the needle's axis would make an angle 35° with the horizontal & the north pole goes down (dips).

3. Horizontal component of earth's magnetic field: ($H$). $H = F \cos \theta$.

Vertical component, $V = F \sin \theta$.

\[ f = \sqrt{V^2 + H^2} \]
\[ \tan \theta = \frac{V}{H} \]

* Neutral Points:
  a) $N$-pole facing North:

b) $N$-pole facing South:

c) $N$-pole facing east:

$F_r$ = resultant force.
Electromagnetism,

- Magnetostatics.

* Electric current & moving charges are source of magnetic field.
* Static charge produces electrostatic field, charge with uniform velocity produces magnetic field, accelerating charge produces electromagnetic field, as well as radiation.

* The north & south poles of a magnet represent the same situation from two opposite sides.

- South pole occurs on that side in which current appears to flow in the wire direction & north pole occur on the other side.

* Strength of electromagnet can be increased by increasing the current, increasing the number of turns in coil or putting a iron core in the coil.
Laplace's Law or Biot-Savart's Law:

If a wire kept in vacuum or air carries a steady current \( I \), then the magnetic field \( dB \) due to a small element of wire of length \( dl \), at a point \( P \) at distance \( r \) depends on \( I, dl, r \) and \( \theta \). \( \theta \) is the angle between the tangent at the midpoint of \( dl \) & line \( OP \).

\[
\vec{d}B = \frac{\mu_0}{4\pi c} \frac{I \cdot \vec{dl} \times \hat{r}}{r^2}
\]

or

\[
\vec{d}B = \frac{\mu_0}{4\pi c} \frac{I \cdot \vec{dl} \times \hat{r}}{r^3}.
\]

\[
\vec{B} = \frac{\mu_0}{4\pi c} \int_{a}^{b} \frac{I \cdot \vec{dl} \times \hat{r}}{r^2}.
\]

- \( I \cdot \vec{dl} \) is the current element.
- Direction of \( \vec{dl} \) is the direction of the current at the location of \( dl \).
- Current element is the original source of the magnetic field.

Application of Biot-Savart Law:

1. \( \vec{B} \) due to a circular current carrying wire:
Magnetostatics.

\[ \vec{B} = \frac{\mu_0}{4\pi} \frac{2\pi a^2 I}{(a^2 + \alpha^2)^{3/2}} \hat{\alpha}. \]

For \( N \) turns of the coil,

\[ \vec{B} = \frac{\mu_0}{4\pi} \frac{2\pi N a^2 I}{(a^2 + \alpha^2)^{3/2}} \hat{\alpha}. \]

At the centre of the coil, (for \( N \) turns)

\[ \vec{B}_0 = \frac{\mu_0}{4\pi} \frac{2\pi N I}{\alpha} \hat{\alpha}. \]

When \( \alpha \gg a \),

\[ \vec{B} = \frac{\mu_0}{4\pi} \frac{2\pi a^2 I}{\alpha^3} \hat{\alpha}. \]

The coil resembles a magnetic dipole.

\[ \vec{B} = \frac{\mu_0}{4\pi} \frac{2\pi a^2 I}{\alpha^3} \hat{\alpha} = \frac{\mu_0}{4\pi} \frac{2N}{\alpha^3} \hat{\alpha}. \]

We get,

\[ \vec{H} = \frac{\mu_0}{\alpha} I \alpha^2 \hat{\alpha} = I A \hat{\alpha}. \]

\( A \rightarrow \) area of the circular turn.

So, a current-carrying loop is equivalent to a magnetic dipole.

Magnetic moment of a loop, carrying current, \( \vec{M} = \text{current} \times \text{Area} \times \hat{\alpha} \).

\( \text{unit} = \text{Am}^2. \)

\[ \vec{M} = NIA \hat{\alpha} \] [for \( N \) turns of coil].
2. Due to a straight wire carrying current \( I \):

\[
B = \frac{\mu_0}{4\pi} \frac{I}{R} (\sin\theta_2 - \sin\theta_1)
\]

[length of wire \( L = R(\tan\theta_2 - \tan\theta_1)\)]

For a wire of infinite length,

\( \theta_2 = +90^\circ, \theta_1 = -90^\circ \)

\[ B = \frac{\mu_0 I}{2\pi} \]

3. \( \vec{B} \) produced by a moving charge:

Magnetic field produced by a single particle of charge \( q \) moving with velocity \( \vec{v} \) at a distance \( r = r \hat{r} \) is

\[ \vec{B} = \frac{\mu_0 q \vec{v} \times \hat{r}}{r^2} \]

* Ampere's Circuital Law: In a magnetic field in empty space the line integral of the magnetic field strength over any closed path \( C \), is equal to the total steady current \( I_c \) enclosed by the closed path multiplied by the permeability \( \mu_0 \) of free space,

\[ \oint_C \vec{B} \cdot d\vec{r} = \mu_0 I_c \]
Magnetostatics.

* Application of Ampere's Circuit Law:

1. Magnetic field due to long straight conductor carrying steady current:
   \[ \vec{B} = \frac{\mu_0}{4\pi} \frac{2I}{r} \hat{\phi} \]

2. \( \vec{B} \) due to a long cylindrical current carrying conductor:
   \[ B = \frac{\mu_0}{4\pi} \frac{2I}{\pi} \left( \frac{r}{R} \right) \quad (r > R) \]
   \[ B = \frac{\mu_0}{4\pi} \frac{2I}{R^2} \left( \frac{r}{R} \right) \quad (r < R) \]

3. \( \vec{B} \) for a solenoid:
   \[ \vec{B} = \mu_0 n I \]
   \[ B = \mu_0 n I \]
   \[ n \rightarrow \text{no. of turns per unit length} \]

The magnetic field is uniform over the entire cross section of the solenoid.

Magnetic field intensity at the two ends of the solenoid:
   \[ B_1 = \frac{1}{2} B = \frac{1}{2} \mu_0 n I \].
4. $\vec{B}$ inside toroid carrying current:

$$B = \mu_0 n i$$

[$n$ = no. of turns per unit length].

There is no field outside & inside space of toroid. The field of toroid is confined completely to the space enclosed by the windings.

* Force on a current carrying wire placed on a magnetic field:

= Motor rule: If there is a magnetic field ($\vec{B}$) & a conductor carrying current ($I$) is placed in it, then the force ($d\vec{F}$) acting on a small element of the conductor is $d\vec{F} = I d\vec{l} \times \vec{B}$.

Force on a wire of finite length.

$$\vec{F} = \int_a^b I d\vec{l} \times \vec{B}.$$  

* Current carrying coil placed in a uniform magnetic field:

Case 1: $\vec{B}$ parallel to coil's plane.

Deflecting torque =

$$\tau = BIA$$

[$A$ = area of coil].
Magneetics

Case 2: \( \vec{B} \) perpendicular to the coil.
\[
\begin{align*}
\vec{f}_1 &= \vec{B} \times \vec{I} \times \vec{l} \\
\vec{f}_2 &= \vec{B} \times \vec{I} \times \vec{b}
\end{align*}
\]
\( \vec{f}_1 \) and \( \vec{f}_2 \) deflecting torque.

Case 3: \( \vec{B} \) makes angle \( \theta \) with normal to the plane of coil.
\[
\begin{align*}
\vec{B}_{\text{normal}} &= \vec{B} \cos \theta \\
\vec{B}_{\text{parallel}} &= \vec{B} \sin \theta \\
\tau &= \vec{B}_{\text{parallel}} \times \vec{I} \times \vec{l} \\
\tau &= \vec{B} \vec{I} \sin \theta \times \vec{l}
\end{align*}
\]
Forces due to \( \vec{B} \), balance themselves.

† Force between two current carrying wires, current - current interaction:

1. Force between two parallel currents:
   Force per unit length \( (F_l) = \frac{\mu_0}{4\pi} \frac{2I_1I_2}{\eta} \)
   (Mutual attraction force).

2. Force between anti-parallel currents:
   \( F_l = \frac{\mu_0}{4\pi} \frac{2I_1I_2}{\eta} \)
   (Mutual repulsion force).
* Force on a charged particle moving in a uniform magnetic field:

If a particle having charge $q$ moves with a velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$, the force $\vec{F}$ acting on it is given by $\vec{F}_m = q\vec{v} \times \vec{B}$.

$\mu_0 = 4\pi \times 10^{-7} \text{ N} \text{A}^{-2}$ or $\text{TM} \text{A}^{-1}$

$[\mu_0] = [\text{MLT}^{-2} \text{A}^{-2}]$

* Lorentz force: If both electric & magnetic fields are present, the force on a charge is $\vec{F}_{\text{Lorentz}} = q\vec{E} + q\vec{v} \times \vec{B}$.

* Magnetic force acting on a moving charge is a no-work force.

* Motion of a charged particle in a magnetic field & cyclotron frequency:

Case 1: Particle having velocity perpendicular to a uniform magnetic field.

$\vec{F}_m = q\vec{v} \times \vec{B} = q\vec{v} \beta \hat{i}$ (centripetal force).
Magnetism.

\[ \frac{mv^2}{r} = qvB \Rightarrow r = \frac{mv}{qB} \]

- \[ T = \frac{2\pi r}{v} = \frac{2\pi m}{qB} \]
- \[ \omega = \frac{qB}{m} \]
- \[ \Theta = \frac{qBt}{m} \]

Case 2: Particle having velocity making an angle \( \Theta \) with \( B \).

\[ B = B \hat{k} \]

Due to \( \sin \Theta \), magnetic force \( qvB \sin \Theta \hat{y} \) act along \( x \) axis.

\[ y \] due to \( \cos \Theta \), the particle proceeds along \( y \) axis.

- \[ r = \frac{mu \sin \Theta}{qB} \]
- \[ \omega = \frac{qB}{2\pi m} \] (no change).

Two motions: i) Uniform circular motion in a plane perpendicular to the field. ii) A uniform motion along the field.

A helix path is followed by the particle.

Pitch of spiral path, \( p = \frac{2\pi r \mu m \cos \Theta}{qB} \).
Through a crossed electric & magnetic field a particle can pass only when velocity of charged particle, $v = \frac{E}{B}$.

Maximum kinetic energy of particle in a cyclotron, $E_{\text{max}} = \frac{q^2 B^2 R^2}{2m}$

$[R \rightarrow \text{largest possible radius}]$.

Force between two moving charged particles:

$$|F| = -\frac{\mu_0}{4\pi} \frac{q_1 q_2}{r^2} v_1 v_2$$

Direction of force is obtained by vectors method.

Magnetic flux:

$$\Phi_m = \int_{S} \mathbf{B} \cdot d\mathbf{S} \quad \text{in Weber.}$$

Hall Effect-1:

$$\mathbf{F}_m = e\mathbf{v}_d \hat{B},$$

$$\mathbf{F}_e = -e\hat{k},$$

Effect of charge accumulation stops when $E = v_d B$.

Tension developed in the wire, $T = 2\pi BR$.  

\[ \text{Diagram} \]