* If a mirror is rotated by an angle \( \theta \) (say anticlockwise), keeping the incident ray fixed then the reflected ray rotates 2\( \theta \) along the same sense i.e. anticlockwise.

\[ \begin{align*}
\theta &\quad \rightarrow \quad M' \\
I &\quad \rightarrow \quad M
\end{align*} \]

* The minimum length of a plane mirror to see one's full length in it is \( \frac{H}{2} \), where \( H \) is the height of man. But, the mirror should be placed in a fixed position.

\[ \begin{align*}
M &\quad \rightarrow \quad x+y \\
\text{Can be placed anywhere between these two lines}
\end{align*} \]

* A man is standing exactly at midway between a wall and a mirror & he wants to see the full height of the wall (behind him) in a plane mirror (in front of him). The minimum height or length of mirror should be \( \frac{H}{3} \), where \( H \) is the height of wall.

\[ \begin{align*}
2x &\quad \rightarrow \quad 2y \\
x+y &\quad \rightarrow \quad x+y
\end{align*} \]
Reflection from a spherical surface:

- **Terminology**
  - Incident
  - Concave mirror.

- Centre (C) of the sphere of which the mirror is a part is called the centre of curvature of the mirror.

- Centre of mirror surface (P) is called the Pole.

  - The line CP is principal axis.
  - AB is the aperture of the mirror.
  - The distance CP is radius of curvature.

**Principal focus:**

In the context of the chapter, rays which are close to the principal axis & make small angles with it, i.e., they are nearly parallel to the axis, are called paraxial rays. Mirrors of small aperture should be restricted to this discussion.
Relation between \( f \) & \( R \): \( f = \frac{R}{2} \)

- Focal length = \( \frac{1}{2} \) (radius of curvature).

- Ray diagram: Four rays can be drawn to construct image of object:
  
  i) Ray 1: Through the centre of curvature, which strikes the mirror normally & reflected back along same path.
  
  ii) Ray 2: Parallel to principal axis; after reflection either actually passes through the principal focus or appears to diverge from it.
  
  iii) Ray 3: Passing through the principal focus \( F \).
  
  iv) Ray 4: Striking at pole \( P \).

Image formed by convex mirror is always virtual, erect & diminished no matter where the object is. Convex mirror gives a wider field of view.

Image formed by concave mirror is erect & virtual when the object is placed between \( F \) & \( P \). In all other positions, image is real.
- **The Mirror Formula:**

  For any mirror, (concave or convex),
  \[
  \frac{1}{v} + \frac{1}{u} = \frac{1}{f}
  \]
  
  \(v \rightarrow \text{distance of image}
  
  \(u \rightarrow \text{distance of object}
  
  \(f \rightarrow \text{focal length}
  
  \(f = \frac{R}{2}\)

- **Magnification:**

  lateral, transverse or linear magnification, \(m\)

  \[m = -\frac{v}{u}\]

  For spherical mirrors \(m = -\frac{v}{u}\). Positive value of \(m\) corresponds to a virtual image (except too). Negative value of \(m\) corresponds to a real, inverted image.

  If for concave mirror,

<table>
<thead>
<tr>
<th>Position of Object</th>
<th>Details of Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>At (\infty)</td>
<td>At (f), real, inverted, (</td>
</tr>
<tr>
<td>Between (C) &amp; (\infty)</td>
<td>Between (F) &amp; (C), real, inverted, (</td>
</tr>
<tr>
<td>At (C)</td>
<td>At (C), real, inverted, (</td>
</tr>
<tr>
<td>Between (F) &amp; (C)</td>
<td>Between (C) &amp; (\infty), real, inverted, (</td>
</tr>
<tr>
<td>At (F)</td>
<td>At infinity, real, inverted, (</td>
</tr>
<tr>
<td>Between (F) &amp; (P)</td>
<td>Behind the mirror, virtual, erect, (</td>
</tr>
</tbody>
</table>
For convex mirror,

<table>
<thead>
<tr>
<th>Position of object</th>
<th>Details of Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>At $\infty$</td>
<td>At $f$, virtual, erect, $1m1 \ll 1$.</td>
</tr>
<tr>
<td>At $(C, P)$ in front of mirror</td>
<td>Between $P$ &amp; $f$, virtual, erect, $1m1 \ll 1$.</td>
</tr>
</tbody>
</table>

### Graph between $\frac{1}{v}$ versus $\frac{1}{u}$:

1. For concave mirror:
   - **Case 1**: Image real - $\frac{1}{v} = -\frac{1}{u} + \frac{1}{f}$
     - Slope of straight line (-1), slope $= \frac{1}{f}$
   - **Case 2**: Image virtual - $\frac{1}{v} = \frac{1}{u} - \frac{1}{f}$
     - Slope (+1), intercept $= -\frac{1}{f}$

2. For convex mirror:
   - Image always virtual.
   - $\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$

* $P$ (dioptric) $= -\frac{1}{f(m)}$.

* Image speed, $V_i = (m^2) (V_o)$.

\[
\frac{dv}{dt} = -\left(\frac{v^2}{u^2}\right) \frac{du}{dt}
\]

* Coordinates of image of a point object if coordinates of object are known:

- Coordinates of object $(x_o, y_o)$
- Coordinates of image $(x_i, y_i)$. 
\[
\begin{align*}
\cdot \alpha_i &= \frac{f x_0}{x_0 - f} \\
\cdot \beta_i &= \frac{f y_0}{f - x_0} \\
\cdot m &= \frac{y_i}{y_0} = -\frac{x_i}{x_0}
\end{align*}
\]

* Longitudinal magnification:

If an object is placed with its length along the principal axis, then

\[
\cdot m_L = \left(\frac{v_2 - v_1}{u_2 - u_1}\right) = -\frac{dv}{du} = \left(\frac{v}{u}\right)^2 = m^2.
\]

\[
\cdot \frac{1}{v^2} + \frac{1}{u^2} = \frac{1}{f}
\]

\[
\Rightarrow -\frac{1}{v^2} \frac{dv}{dt} - \frac{1}{u^2} \frac{du}{dt} = 0.
\]

\[
\Rightarrow \frac{dv}{dt} = -\left(\frac{u^2}{v^2}\right) \frac{du}{dt}
\]

* Component of incident ray along the inside normal of a plane mirror gets reversed while the component of ray perpendicular to it remains unchanged.

eq. If incident ray is \((\hat{i} + \hat{j} - \hat{k})\) & normal on incident point is \((\hat{i} + \hat{j})\) then reflected ray \(\overrightarrow{(-\hat{i} - \hat{j}) - \hat{k}}\), reversing \((\hat{i} + \hat{j})\).
1. Deviation:

\[ \delta = 180^\circ - 2\theta \]

- If two plane mirrors are inclined to each other at an angle of 90°, the emergent ray is antiparallel to incident ray, if it reflects once from each mirror, whatever be the angle of incidence.

2. Image by two inclined plane mirrors:

When two plane mirrors are inclined to each other at an angle \( \theta \), then number of images (\( n \)) formed of an object which is kept between them:

i) If \( \frac{360^\circ}{\theta} = \text{even integer} \), then \( n = \left( \frac{360^\circ}{\theta} - 1 \right) \).

ii) If \( \frac{360^\circ}{\theta} = \text{odd integer} \),

a) Object placed symmetrically,

\[ n = \frac{360^\circ}{\theta} - 1 \]

b) Object placed asymmetrically,

\[ n = \frac{360^\circ}{\theta} \]

3. Movement of object:

i) When the object moves with speed \( u \) towards (or away) from the plane mirror then image also moves towards (or away) with speed \( u \). But, relative speed of image wrt object is \( 2u \).

ii) When mirror moves towards the stationary object with speed \( u \), the image will move with speed \( 2u \).

4. For watch (clock) related problems:

Actual time = 11:60 - given time

(When seen through a mirror)
Uses of mirrors:

i) Concave: Shaving mirror, search light, cinema projector, telescope, by ENT specialists.

ii) Convex: Road lamps, side mirror in vehicles.

Important graphs:

a) Graph between \( u \) vs \( v \) for real image of concave mirror:

\[
\begin{array}{c}
\text{f} \\
\text{2f} \\
\text{f} \\
\text{2f} \\
\text{u} \\
\end{array}
\]

b) Graph between \( u \) & \( m \) for virtual image by concave mirror:

\[
\begin{array}{c}
\text{m} \\
\text{f} \\
\text{u} \\
\end{array}
\]

c) Graph between \( u \) & \( m \) for virtual image by convex mirror:

\[
\begin{array}{c}
\text{m} \\
\text{u} \\
\end{array}
\]

Focal length of a mirror is independent of material of mirror, medium in which it is placed, wavelength of incident length.

Divergence or convergence power of mirror doesn’t change with change in medium.
If an object is moving at a speed \( v_0 \) towards a spherical mirror along \( \theta \) axis, the speed of image away from mirror is

\[
\nu_i = - \left( \frac{f}{u-f} \right)^2 v_0 \quad \text{(with sign convention)}
\]

When object is moved from focus to \( \infty \) at constant speed, the image will move faster in the beginning & slower later on.

Conver mirror forming real images:

\[
u < f
\]

Refraction of Light:

- Laws of Refraction:
  1. Snell's Law: For two particular media, the ratio of sini of the angle of incidence to the sini of the angle of refraction is constant.

\[
\frac{\sin i_1}{\sin i_2} = \text{constant}
\]

2. The incident ray, reflected ray & the refracted ray are all lie in the same plane.

The constant ratio \( \frac{\sin i_1}{\sin i_2} \) is called the refractive index for light passing from the first to the second medium (2 with respect to 1).

\[
\frac{1}{\mu_2} = \frac{\sin i_1}{\sin i_2}
\]
A ray of light moves from medium 1 to medium 2. Its frequency remains constant, but its wavelength changes.

As a ray of light moves from medium 1 to medium 2, its wavelength changes. According to Snell's Law:

\[
\frac{\sin \theta_1}{\sin \theta_2} = \frac{\mu_2}{\mu_1}
\]

where \(\theta_1\) and \(\theta_2\) are the angles of incidence and refraction, respectively, and \(\mu_1\) and \(\mu_2\) are the refractive indices of the two media.

The refractive index of a medium with respect to air (vacuum) is given by:

\[
\frac{1}{\mu} = \text{refractive index}
\]

For air to medium, \(\mu = 1\).
- Single Refraction from a Plane Surface:
  i) If a point is at a depth $d$ from a water surface, then
  \[ d_{\text{apparent}} = \frac{d_{\text{actual}}}{\mu} = \frac{d}{\mu} \]

  ii) \( AI = \mu (AO) \)

  iii) \( AI = \frac{AO}{\mu} \)

- Shift due to a glass slab:
  i) Normal Shift
  \[ OI = (1 - \frac{1}{\mu e}) t \]
ii) Lateral shift

\[ d = \left[ 1 - \frac{\cos \theta}{\sqrt{\mu^2 - \sin^2 \theta}} \right] \tan \theta \]

for small angle of incidence,

\[ d = ti \left( \frac{\mu - 1}{\mu} \right). \]

• Refraction from a spherical surface:

\[ \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \]

/ Lateral magnification: \( m = \frac{\mu_1}{\mu_2} \cdot \frac{v}{u} \)

/ Thin Lenses:

/ First Focus: A point at which if an object is placed, the image of the object is formed at infinity.

/ Second Focus or Principal focus:

Narrow beam of light travelling parallel to principal axis either converge or diverge at a point; this point is \( F_2 \).
Lens Maker's Formula: \[
\frac{1}{f} = \frac{\mu_2 - \mu_1}{\mu_1} \left( \frac{1}{R_1} - \frac{1}{R_2} \right)
\]
\[
\frac{1}{f} = (\mu_2 - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)
\]
\[
\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{[lens formula]}
\]

For a converging lens, \( f \) is +ve.
For a diverging lens, \( f \) is -ve.

If a lens is immersed in some liquid, focal length of lens would change.
\[
f_{\text{water}} = f_{\text{air}} \quad \text{if} \quad \mu_g = \frac{3}{2}, \quad \mu_w = \frac{4}{3}
\]

Magnification, \( m = \frac{\text{height of image}}{\text{height of object}} = \frac{v}{u} \).
+ve value of \( m \) corresponds to a virtual image.
-ve value of \( m \) corresponds to a real image.
Displacement method to determine focal length of a convex lens: \((d > 4f)\).

\[
\begin{align*}
\text{Screen} & \quad u^2 - du + df = 0 \\
\Rightarrow u & = \frac{d \pm \sqrt{d(d-4f)}}{2}.
\end{align*}
\]

1) When \(d = 4f\), \(u = 2f\).

Minimum distance between an object & its real image in case of convex lens is \(4f\).

2) When \(d > 4f\), two possibilities -
\[
u = \frac{d + \sqrt{d(d-4f)}}{2} \quad \text{&} \quad u = \frac{d - \sqrt{d(d-4f)}}{2}.
\]

(Corollary of (2):
3) If \(I_1\) is the image length in one position of the object & \(I_2\) is the image length in second position, then object length \(O\) is given by
\[
0 = \sqrt{I_1 I_2}.
\]

Focal length of two or more thin lenses in contact:

Equivalent focal length being \(F\),
\[
\frac{1}{F} = \sum_{i=1}^{n} \frac{1}{f_i}
\]
Power of an Optical Instrument:

For a lens,

\[ P \ (\text{in dioptre}) = \frac{1}{f \ (\text{metre})} \]

[If lens converges the rays parallel to principal axis its power is +ve & if diverges then -ve.]

\[ F_1 < F_2 \Rightarrow P_1 > P_2 \]

more converging \( F_1 \)

more diverging \( F_2 \)

* Total Internal Reflection:

Critical angle,

\[ \theta_c = \sin^{-1} \left( \frac{\mu_R}{\mu_D} \right) \]

If \( \theta > \theta_c \), then no refracted beam is observed & incident beam is completely reflected.

For, \( \mu_R = 1 \), \( \mu_D = \mu \) \( \Rightarrow \) \( \theta_c = \sin^{-1} \left( \frac{1}{\mu} \right) \).

\[ \text{Totally reflected prisms:} \]

For glass \( (\mu = \frac{3}{2}) \), \( \theta_c = \sin^{-1} \left( \frac{1}{\mu} \right) = 42^\circ \).

When incidence angle is greater than \( 42^\circ \), it is totally reflected.
Optical fibres: When incidence angle is greater than the $\theta_c$ of glass, then light reflects internally without loss.

Refraction through Prism:
1. $r_1 + r_2 = A$
2. $\delta = (i_1 + i_2) - A$
2.1. $\delta = (\mu - 1) A$ \[\text{if } A \ll \mu \ll i_1 \text{ are small} \]
3. For minimum deviation, $i_1 = i_2 = i$
\[i = \frac{A + \delta_{\text{min}}}{2}, \quad \mu = \frac{\sin\left(\frac{A + \delta_{\text{min}}}{2}\right)}{\sin\left(\frac{A}{2}\right)}\]
4. Condition for no emergence: $A > 2\theta_c$
5. If $\delta_r, \delta_y, \delta_v$ are the deviations for red, yellow & violet components of white light, then average deviation is measured by $\delta_y$. $\delta_v - \delta_r$ is called angular dispersion.

The dispersive power ($\omega$) of a material is defined as the ratio of angular dispersion to the average deviation.

\[\omega = \frac{\mu_v - \mu_r}{\mu_y - 1}\]
6. Dispersion without average deviation:
\[
\frac{A}{A'} = \frac{\mu y' - 1}{\mu y - 1}.
\]

\[S_v - S_r = (\mu y - 1) A (\omega - \omega').\]

7. Average deviation without dispersion:
\[
\frac{A}{A'} = \frac{(\mu y - 1) \omega'}{(\mu y - 1) \omega} = \frac{\mu y' - \mu y}{\mu y' - \mu y'}.
\]

\[S_y = (\mu y - 1) A \left( 1 - \frac{\omega}{\omega'} \right).\]

* If a beaker contains various immiscible liquids,

Apparent depth of bottom =
\[
\frac{d_1 + d_2 + d_3 + \ldots}{\mu_1 + \mu_2 + \mu_3 + \ldots}
\]

\[\mu_{\text{combination}} = \frac{d_{\text{actual}}}{d_{\text{app}}} = \frac{d_1 + d_2 + \ldots}{\frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \ldots}.
\]

* Simple Microscope:

![Diagram of a simple microscope](image-url)
Magnification:

Angular magnification, \( m = \frac{D}{u} = \frac{D}{f} + \frac{D}{v} \)

\( D \rightarrow \) least distance of distinct vision, for normal eye 25 cm
[All quantities are only their magnitudes]

Two methods of using magnifying lens:

1. Image at the near point: \( v = D \).
   \[
   m_D = 1 + \frac{D}{f}
   \]

2. Image at far point: \( v = \) nearing infinity
   \[
   m_\infty = \frac{D}{f}
   \]

If lens is kept at a distance ‘a’ from the eye, then

\[
\begin{align*}
  m_D &= 1 + \frac{D-a}{f} \\
  m_\infty &= \frac{D-a}{f}
\end{align*}
\]
* Compound Microscope: \( O \rightarrow \) Objective lens, \( E \rightarrow \) eyepiece

\[ f_O < f_E \]

\[ u_o \quad v_o \quad u_e \quad v_e = D \text{ to } \infty \]

- **Magnification:**
  - **Distinct vision:**
    \[ m_D = -\frac{v_o}{u_o} \left(1 + \frac{D}{f_e}\right) \quad m_D = -\frac{L}{f_o} \left(1 + \frac{D}{f_e}\right) \]
  - **Normal vision:**
    \[ m_{\infty} = -\frac{v_o}{u_o} \frac{D}{f_e} \quad m_{\infty} = -\frac{L}{f_o} \frac{D}{f_e} \]

- **Length of tube:**
  - i) When final image at \( D \),
    \[ L_D = v_o + u_e = \frac{u_o f_o}{u_o - f_o} + \frac{f_e D}{f_e + D} \] [Not to use sign convention!]

  - ii) When final image at \( \infty \),
    \[ L_{\infty} = v_o + f_e = \frac{u_o f_o}{u_o - f_o} + f_e \]

**If objective & eyepieces are interchanged** there is no change in magnification.
* Astronomical Telescope: Refracting:

\[ f_0 > f_e \]

![Diagram of astronomical telescope focusing](image)

\[ m = - \frac{f_0}{u_e} \]

- Magnification:

i) Normal focusing: Final image is at infinity, so that the eye can see the image without accommodation. Here, \( u_e = f_e \).

\[ m_\infty = - \frac{f_0}{f_e} ; \quad L_\infty = f_0 + f_e \]

ii) Distinct focusing: Final image is formed at the least distance of distinct vision. Here, \( u_e = -D \).

\[ m_D = - \frac{f_0}{f_e} \left( \frac{1}{f_e} + \frac{1}{D} \right) ; \quad L_D = f_0 + u_e \]

\[ = - \frac{f_0}{f_e} \left( 1 + \frac{f_e}{D} \right) \]

iii) In general case, magnification,

\[ m_v = - \frac{f_0}{u_e} \]
**Astronomical Telescope - Reflecting**

- MN at 45° to the axis of paraboloidal mirror.
- Final image at infinity.

- Magnification, \( m = \frac{\text{focal length of objective (f)}}{\text{focal length of eye-piece (f)}} \)

* For a telescope, with increase in length of the telescope, magnification decreases.

\[ \text{Resolving power of a telescope} \propto \frac{D}{\lambda} \]

So, large aperture increases resolving power.

If 4 convex lens of focal lengths \( f_1 > f_2 > f_3 > f_4 \) are given, ideal combination for-

1) a good microscope \( \Rightarrow f_4 \) as objective lens, \( f_3 \) as eye-piece.

2) a good telescope \( \Rightarrow f_1 \) as objective, \( f_4 \) as eye-piece.
Resolving Limit (RL) & Resolving Power (RP):

i) For a microscope: Minimum distance between two lines at which they are just distinct is called RL and its reciprocal is RP.

\[
RL = \frac{\lambda}{2\mu \sin \theta} \quad \text{and} \quad RP = \frac{2\mu \sin \theta}{\lambda}
\]

- \(\lambda\) \rightarrow \text{wavelength of light used to illuminate the object.}
- \(\mu\) \rightarrow \text{refractive index between object and objective medium.}
- \(\theta\) \rightarrow \text{half angle of the cone of light from the point object.}
- \(\mu \sin \theta\) \rightarrow \text{numerical aperture.}

ii) For a telescope: Smallest angular separation (d\(\theta\)) between two distant objects whose images are separated in a telescope, is called RL.

\[
RL = d\theta = \frac{1.22 \lambda}{a}
\]

\[
RP = \frac{1}{d\theta} = \frac{a}{1.22 \lambda}
\]

- \(\lambda\) \rightarrow \text{wavelength of light.}
- \(a\) \rightarrow \text{aperture of objective.}

- Minimum separation (d) between objects, so they can just be resolved by a telescope is

\[
d = \frac{\rho}{RP} = \frac{1.22 \lambda \rho}{a}
\]

- \(\rho\) \rightarrow \text{distance of objects from telescope.}
<table>
<thead>
<tr>
<th>Instrument</th>
<th>( m )</th>
<th>( m_{\text{general}} )</th>
<th>( m_D )</th>
<th>( m_{\infty} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Simple microscope</td>
<td>( \frac{D}{v} + \frac{D}{f} )</td>
<td>( 1 + \frac{D}{f} )</td>
<td>( \frac{D}{f} )</td>
<td></td>
</tr>
<tr>
<td>2. Compound microscope</td>
<td>(- \frac{v_0}{u_0} \cdot \frac{D}{u_2} )</td>
<td>(- \frac{v_0}{u_0} \left(1 + \frac{D}{f_{\infty}}\right))</td>
<td>(- \frac{v_0}{u_0} \cdot \frac{D}{f_{\infty}})</td>
<td>(- \frac{L}{f_0} \cdot \frac{D}{f_{\infty}})</td>
</tr>
<tr>
<td>3. Astronomical Telescope</td>
<td>(- \frac{f_0}{u_2} )</td>
<td>(- \frac{f_0}{f_{\infty}} \left(1 + \frac{f_{\infty}}{D}\right))</td>
<td>(- \frac{f_0}{f_{\infty}})</td>
<td></td>
</tr>
</tbody>
</table>

* \( \text{RP}_{\text{microscope}} = \frac{2 \mu \sin \theta}{\lambda} \)

* \( \text{RP}_{\text{telescope}} = \frac{a}{1.22 \lambda} \)
Physical Optics:

YDSE:
1) For bright fringes, \( y = \frac{n \lambda D}{d} \) \([n = 0, \pm 1, \pm 2, \ldots]\)
2) For dark fringes, \( y = \frac{(2n+1) \lambda D}{2d} \) \([n = 0, \pm 1, \pm 2, \ldots]\)
3) Fringe width, \( w = \frac{\lambda D}{d} \)
4) \( \Delta x = d \sin \theta = \frac{yd}{D} \)
5) \( \phi = \frac{2 \pi c}{\lambda} \Delta x \)
6) \( I = 4I_0 \cos^2 \frac{\phi}{2} \)
7) \( \Delta \psi = \Delta \lambda \)
8) Path difference by \( \Delta \)

\[ \text{Slab} = (\mu - 1) t \]
\[ \text{Vertical shift} = \frac{\mu - 1 + D}{d} \]

No of fringes shifted = \( \frac{(\mu - 1) t}{\lambda} \)

SSE:
1) Width of central maxima = \( \frac{2 \lambda D}{d} \)
2) \( n \)-th secondary maxima, \( \Delta x = (2n+1) \frac{\lambda}{2} \)
3) \( n \)-th secondary minima, \( \Delta x = n \lambda \)

\[ p \tan \theta = \mu \]

\[ I_{\text{max}} = 4I_0 \]

**Example:**
11-th bright fringe = \((y_{11})_{\text{br}}\) \[ \frac{11 \lambda D}{d} \]

11-th dark fringe = \((y_{11})_{\text{dark}}\) \[ \frac{(2 \times 11 + 1) \lambda D}{2d} \]

Breasted's Law:** 
\( n \tan \theta = \mu \)