### Units & Measurements

#### Physical Quantities

<table>
<thead>
<tr>
<th>Constant or Ratio</th>
<th>Scalars (charge, mass)</th>
<th>Vectors (force, velocity)</th>
<th>Phasors (have magnitude &amp; phase)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(refractive index, specific gravity)</td>
<td>have magnitude &amp; units may have direction, vector laws not applicable.</td>
<td>have magnitude, unit, direction; follow triangle law of addition</td>
<td>follow triangle law. AC voltage, AC current</td>
</tr>
</tbody>
</table>

**Conversion Factors:**

In a wave \( y = x \sin(\omega t - kx) \), \( k \) is a conversion factor. In frequency modulation, \( kF \omega f_2 = \delta \), \( k \) converts voltage into angle, so a conversion factor.

**Tensors:**

Don't have specified direction, but have different values in different directions. (Moment of inertia is a tensor). In anisotropic media, density, dielectric constant, stress become tensors. Scalar - a tensor of zero rank; vector - a tensor of rank one to four.

### Representation of Physical Quantity:

- \( n_1 u_1 = n_2 u_2 \) (\( n_i \rightarrow \) numerical value of physical quantity in proper unit \( u_i \)).
- Two or more physical quantities are added or subtracted when their units & dimensions are same.
- After multiplication or division, the resultant quantity may have different unit.

### Units:

- i) Fundamental
- ii) Derived
- iii) Supplementary
- iv) Practical

### Fundamental Units:

Independent units. One fundamental unit cannot be expressed in the form of other fundamental units.
ii) Derived Unit: Dependent Units. Derived from fundamental units.

iii) Supplementary: Unit having no dimensions, eg. Plane angle → radian
     Solid angle → steradian

iv) Practical Units: Neither fundamental nor derived.

  a) 1 fermi = 1 fm = $10^{-15}$ m,
  b) 1 X-Ray Unit = $10^{-13}$ m,
  c) 1 Å = $10^{-10}$ m,
  d) 1 pm = 1 micron = $10^{-6}$ m,
  e) 1 AU = $1.49 	imes 10^{11}$ m,
  f) 1 ly = $9.46 	imes 10^{15}$ m,
  g) 1 parsec = 1 pc = $3.08 	imes 10^{16}$ m = 3.26 ly,
  h) 1 shake = $10^{-8}$ sec,
  i) 1 metric ton = 1000 kg,
  j) 1 pound = 453.6 gram weight,
  k) 1 atm = $1.013 	imes 10^5$ N/m$^2$ = 1 bar,
  l) 1 torr = 1 mmHg = 133.3 N/m$^2$,
  m) 1 mile = 1.6 km,
  n) 1 yard = 3 ft,
  o) 1 ft = 12 inch,
  p) 1 inch = 2.54 cm,
  q) 1 Chandrasekhar limit = 1.4 x mass of the sun = $2.8 	imes 10^{30}$ kg; if mass of an object becomes this, under gravitational collapse it turns out to be a white dwarf.

Abbreviations:

<table>
<thead>
<tr>
<th>Factor</th>
<th>Prefix</th>
<th>Symbol</th>
<th>Factor</th>
<th>Prefix</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{34}$</td>
<td>yotta</td>
<td>Y</td>
<td>$10^6$</td>
<td>kilo</td>
<td>k</td>
</tr>
<tr>
<td>$10^{24}$</td>
<td>zetta</td>
<td>Z</td>
<td>$10^5$</td>
<td>hecto</td>
<td>h</td>
</tr>
<tr>
<td>$10^{18}$</td>
<td>exa</td>
<td>E</td>
<td>$10^4$</td>
<td>deka</td>
<td>da</td>
</tr>
<tr>
<td>$10^{15}$</td>
<td>peta</td>
<td>P</td>
<td>$10^3$</td>
<td>deci</td>
<td>d</td>
</tr>
<tr>
<td>$10^{12}$</td>
<td>tera</td>
<td>T</td>
<td>$10^2$</td>
<td>centi</td>
<td>c</td>
</tr>
<tr>
<td>$10^{12}$</td>
<td>trillion</td>
<td></td>
<td>$10^1$</td>
<td>milli</td>
<td>m</td>
</tr>
<tr>
<td>$10^9$</td>
<td>giga</td>
<td>G</td>
<td>$10^{-1}$</td>
<td>micro</td>
<td>µ</td>
</tr>
<tr>
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<td>tera</td>
<td>T</td>
<td>$10^{-2}$</td>
<td>nano</td>
<td>n</td>
</tr>
<tr>
<td>$10^6$</td>
<td>mega</td>
<td>M</td>
<td>$10^{-3}$</td>
<td>pico</td>
<td>p</td>
</tr>
<tr>
<td>$10^4$</td>
<td>kina</td>
<td>K</td>
<td>$10^{-4}$</td>
<td>femto</td>
<td>f</td>
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<tr>
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<td>m</td>
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<td>a</td>
</tr>
<tr>
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<td>micro</td>
<td>µ</td>
<td>$10^{-6}$</td>
<td>zepto</td>
<td>z</td>
</tr>
<tr>
<td>$10^1$</td>
<td>nano</td>
<td>n</td>
<td>$10^{-7}$</td>
<td>yocto</td>
<td>y</td>
</tr>
</tbody>
</table>
* Radius of earth - $6.4 \times 10^6$ m
* Mass of Sun - $2 \times 10^{30}$ kg
* Mass of moon - $7 \times 10^{22}$ kg.

* Dimensions & Power raised to fundamental units to get derived unit.

* Homogeneity Principle: If the dimensions of left hand side of an equation are equal to the dimensions of the right hand side of the equation, then the equation is dimensionally correct.

* Converting physical quantity from one system to another:
  \[ n_1 [u_1] = n_2 [u_2] \]
  \[ \Rightarrow n_1 \left[ \frac{M_1}{L_1^b T_1^c} \right] = n_2 \left[ \frac{M_2}{L_2^b T_2^c} \right] \]
  \[ \Rightarrow n_2 = n_1 \left[ \frac{M_1}{M_2} \right]^a \left[ \frac{L_1}{L_2} \right]^b \left[ \frac{T_1}{T_2} \right]^c \]

* Significant figures rules:
  a) All non-zero digits are significant.
  b) A zero becomes significant if between two non-zero digits.
  c) Leading zeros or the zeros placed to the left of the number are never significant.
  d) Trailing zeros or the zeros placed to the right of the number are significant.
  e) In exponential notation the numerical portion gives the number of significant figures.
Rounding off: 1) digit to be dropped less than 5, preceding digit left unchanged.
2) more than 5, preceding digit raised by 1.
3) equals 5, followed by digits other than zero, preceding digit raised by 1. 4) equals 5, followed by zeroes, preceding digit being even or odd, the digit is left unchanged or raised by one respectively.

Addition - Subtraction rule: All measured values are rounded off to smallest number of decimal places.

Multiplication - Division rule: answer should be in the form of least number of significant figures.

Error on measurement: \( y_1, y_2, \ldots, y_n \) are results for \( n \) times experiments,

- True value, \( y_{\text{mean}} = \frac{\sum_{i=1}^{n} y_i}{n} \)
- Order of error \( \pm \sigma = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (y_i - \bar{y})^2} \)

Value of quantity \( y_{\text{mean}} \pm \sigma \)

- Absolute error: measured values \( a_1, a_2, \ldots, a_n \)
  \[ a_{\text{mean}} = \frac{a_1 + a_2 + \ldots + a_n}{n} \]
  \[ \Delta a_1 = a_{\text{mean}} - a_1 \]
  \[ \Delta a_n = a_{\text{mean}} - a_n \] — absolute error.
Units & Measurements.

- Mean absolute error: $\overline{\Delta a}$

$$\overline{\Delta a} = \frac{1 \Delta a_1 + 1 \Delta a_2 + \ldots + 1 \Delta a_n}{n}.$$  

Final result, $a = a_{\text{mean}} \pm \overline{\Delta a}$

- Relative error / Fractional error:

$$\frac{\Delta a}{a_{\text{mean}}}$$

- Percentage error:

$$\frac{\Delta a}{a_{\text{mean}}} \times 100\%$$

Properties:

a) $x = a + b$.

$$\Delta x = \pm (\Delta a + \Delta b).$$

$$\% \text{ error} = \frac{(\Delta a + \Delta b)}{a + b} \times 100\%$$

b) $x = a - b$.

$$\Delta x = \pm (\Delta a + \Delta b)$$

$$\% \text{ error} = \frac{(\Delta a + \Delta b)}{\Delta a \Delta b} \times 100\%$$

c) $x = a \times b$.

$$\frac{\Delta x}{a} = \pm \left( \frac{\Delta a}{a} + \frac{\Delta b}{b} \right).$$

$\% \text{ error in } x = \% \text{ error in } a + \% \text{ error in } b.$

d) $x = a / b$.

$$\frac{\Delta x}{x} = \pm \left( \frac{\Delta a}{a} + \frac{\Delta b}{b} \right).$$
\( e) \quad x = \frac{a^n}{b^m} \)

\[ \frac{\Delta x}{x} = \pm \left( n \frac{\Delta a}{a} + m \frac{\Delta b}{b} \right). \]

\( b) \quad \text{i.e., equal to} \quad x = n \left( \% \text{ error in } a \right) + m \left( \% \text{ error in } b \right), \]